CHAPTER 15

Dynodic Efficiency of Beta Emitters

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INTRODUCTION

In liquid scintillation counting measurements[1-3] of beta emitters, contrasted models give us efficient detection through a study of internal processes in the vial and the response of the photocathode. Many of these models use a figure of merit of the system as a parameter.[4-8]

Other models explain the dynodes response in a photomultiplier as a set of electrons reaching the first dynode. In this way we can get the electron distribution at the output of any dynodic stage in the photomultiplier.[9-12]

For each electron set that enters the first dynode, however, there is always a nonzero probability of nondetection in the dynodes. This leads to a lower efficiency at the dynode output than at the photocathode.

The present paper has studied the efficiency loss through the dynodic stages for ^{3}H and ^{14}C and applied it to the standardization of ^{14}C via the efficiency tracing method.[7-8] First, a model for the electron multiplication loss is described, then the efficiency loss through dynodes is computed for ^{3}H and ^{14}C, for a single tube and for two tubes in coincidence.

Experimental quench curves for ^{14}C have been compared to the theoretical predictions.

NON DETECTION PROBABILITY MODEL

It is assummed that the electron production process, at the output of the photocathode, is governed by the Poisson statistics. If **N** electrons are expected in the average, the probability of obtaining n electrons is:

$$P(n) = \frac{\bar{N}^{n}\, e^{-\bar{N}}}{n!} \qquad (1)$$

The number **n** of electrons that leaves the photocathode is multiplied through sucessive dynodes, the amplification being typified by means of a free parameter, **G**, the dynodic gain. This parameter is defined as the number of electrons that come out from a dynode when a single electron comes into the dynode.

Statistically, there is a nonzero probability of having no electrons at the output of the Kth dynodic stage when **n** electrons arrive at the first dynode. This nonzero probability follows this iterative expression:[12]

$$P_K(n,0) = \exp(-G \cdot n) \exp[G \cdot n \cdot P_{K-1}(1,0)]; \quad K > 1 \tag{2}$$

$$P_K(n,0) = \exp(-G \cdot n) ; \quad K = 1 \tag{3}$$

where n = the number of electrons that leaves the photocathode
K = the order of dynodic stages
G = the dynodic gain

The nondetection probability leads to a dynodic detection efficiency lower than the photocathode efficiency, because when **n** electrons leave the photocathode with probability 1, the total sum of the emission probabilities at the Kth dynodic output is $1 - P_K(n,0)$.

DETECTION EFFICIENCY

Single photomultiplier tube

Beta emitters have an emission spectra given by the Fermi distribution:

$$N_i(E) = C \cdot F(Z,E) \cdot (E_i - E)^2 (E + 1)(E^2 + 2E)^{1/2} \tag{4}$$

where $N_i(E)$
= the number of particle beta with energies between E, E + dE
E_i = the maximum kinetic energy of beta particles in the ith energy band
C = the shape factor[13]
F(Z,E) = the relativistic expression of the Coulomb factor[14] (all in m_0c^2 units)

When a radionuclide is dissolved in a scintillation liquid, its beta particles transfer almost all their energy to the scintillator, that is, convert to light. This light emission is partially detected by the photocathode of the photomultiplier tube. The number of electrons that leave the photocathode is a function of the beta particle energy.

The average number of electrons that can be expected when a beta particle is emitted in the energy band (E, E + dE), is:

$$N(E) = \frac{E \cdot Q(E)}{M} \tag{5}$$

where E = the beta particle energy
Q(E) = the ionization quench factor
M = the figure of merit, or energy necessary to produce an electron at the photocathode

A good aproximation for Q(E) is Equation 7:

$$Q(E) = 1 - 0.9624\ e^{-0.5457};\ E > 10\ \text{keV} \tag{6}$$

$$Q(E) = 0.1253\ \text{Ln}(E) + 0.4339;\ 0.1\ \text{keV} < E < 10\ \text{keV} \tag{7}$$

According to that, mean value r electrons could be obtained with probability:

$$P(r,E) = \frac{N(E)^r\ e^{N(E)}}{r!} \tag{8}$$

The detection efficiency at the output of the photocathode for a single tube and for a unique beta particle in the ith band is:

$$E_{f1} = \sum_r P(r,E) \tag{9}$$

For all the particles in the ith energy band, the probability for having electrons is:

$$P_i(r) = P(r,E)\ N_i(E) \tag{10}$$

and the detection efficiency for the ith energy band at the output of the photocathode is:

$$E_{f1} = \sum_r P_i(r) \tag{11}$$

Therefore the efficiency for the whole spectrum at the output of the photocathode for a single photomultiplier tube is:

$$E_f = \sum_i E_{fi} \tag{12}$$

Now the detection efficiency for a beta emitter of the Kth dynodic stage output of a single photomultiplier tube can be analyzed. When a beta particle in the ith energy band is emitted, the r electrons that leave the photocathode with probability P(r,E) will have a nonemission probability at the Kth dynode given by:

$$P_{K1}(r,0) = P_K(r,0)\ P(r,E) \qquad (13)$$

and summing up for all the electrons that leave the photocathode when a beta particle in the ith energy band is emitted, we arrive at:

$$P_{K1}(0) = \sum_{r} P_K(r,0)\ P(r,E) \qquad (14)$$

Therefore, the efficiency at the output of the Kth dynode for this single beta particle in the ith band is:

$$E_{K1} = \left(\sum_{r} P(r,E)\right) - P_{K1}(0) \qquad (15)$$

For the $N_i(E)$ particles in the ith band, the nondetection probability at the output of the Kth dynode is:

$$P_{Ki}(0) = P_{K1}(0)\ N_i(E) \qquad (16)$$

and the efficiency of detection for this band will be:

$$E_{Ki} = E_{fi} - P_{Ki}(0) \qquad (17)$$

Finally the detection efficiency of the Kth dynodic stage output of a single photomultiplier tube for the whole beta emitter spectrum is:

$$E_K = \sum_{i} E_{Ki} \qquad (18)$$

Two Photomultiplier Tubes in Coincidence

The previous procedure can be easily extended for a system composed of two photomultiplier tubes in coincidence. Both tubes are assummed to be identical so the detection efficiency at the output of the photocathodes for a beta particle emitted in the ith-energy band for that coincidence system is:

$$E'_{f1} = \left(\sum_{r} P(r,E)\right)^2 \qquad (19)$$

and for $N_i(E)$ particles in the energy band we will have:

$$E'_{fi} = E'_{f1} \cdot N_i(E) \qquad (20)$$

Therefore the photocathode efficiency for the whole electron spectrum at the output of the system of two tubes in coincidence is:

$$E'_f = \sum_i E'_{fi} \tag{21}$$

Now the detection efficiency at the output of the Kth dynode for two tubes in coincidence can be analyzed. For a single beta particle in the ith energy band the detection efficiency is:

$$E'_{K1} = (E_{K1})^2 \tag{22}$$

and for all electrons in the ith energy band we will have:

$$E'_{Ki} = E'_{K1} \cdot N_i(E) \tag{23}$$

Finally, the detection efficiency for a beta spectrum at the output of the Kth dynodic stages for two tubes in coincidence will be:

$$E'_K = \sum_i E'_{Ki} \tag{24}$$

RESULTS

Non detection probability

The nondetection probability has been studied for the following parameters:

The dynodic gain varying, from 1 to 10 with step 1, the number of dynodic stages 12, and the number of electrons injected into the first dynode, from 1 to 19 with step increment 2.

Table 1 shows the nondetection probability for the 12 dynodic stages and for the 1, 3, 5, 7, 11, and 19 electrons injected into the first dynode. The dynodic gain is 1 or 2.

In Figure 1a, the nondetection probability is plotted vs the dynodic stage. For 1, 5, 11, or 19 electrons injected and a dynodic gain 1, in Figure 1b, the nondetection probability is plotted vs dynodics gain.

Detection Efficiency for Beta Emitters

Two different photomultiplier tubes with 12 dynodes have been simulated:

1. low gain, G has a value of 7 in the first dynode and a value of 2 in the others.
2. high gain, G has a value of 30 in the first dynode and a value of 3 in the others.

Table 2 gives the efficiency at the output of the 12th dynode for ^3H, for high gain and two photomultiplier tubes in coincidence.

Table 1. Nondetection Probability in Successive Dynodic Stages and Different Number of Electrons Injected into the First Dynode

STAGE	1 ELECT.	3 ELECT.	5 ELECT.	7 ELECT.	11 ELECT.	19 ELECT.
01	0.37	0.05	0.01	0.00	0.00	0.00
02	0.53	0.15	0.04	0.01	0.00	0.00
03	0.63	0.24	0.10	0.04	0.00	0.00
04	0.69	0.32	0.15	0.07	0.01	0.00
05	0.73	0.39	0.21	0.11	0.02	0.00
06	0.76	0.45	0.26	0.15	0.03	0.01
07	0.79	0.49	0.31	0.19	0.05	0.01
08	0.81	0.53	0.35	0.23	0.07	0.02
09	0.83	0.57	0.39	0.26	0.10	0.03
10	0.84	0.60	0.42	0.30	0.13	0.04
11	0.85	0.62	0.45	0.33	0.15	0.05
12	0.86	0.64	0.48	0.36	0.17	0.06
			DYNODIC GAIN: 1			
01	0.13	0.00	0.00			
02	0.17	0.01	0.00	Nondetection probability < 1.E-4		
03	0.19	0.01	0.00			
04	0.20	0.01	0.00			
05	0.20	0.01	0.00			
06	0.20	0.01	0.00			
07	0.20	0.01	0.00			
08	0.20	0.01	0.00			
09	0.20	0.01	0.00			
10	0.20	0.01	0.00			
11	0.20	0.01	0.00			
12	0.20	0.01	0.00			

Figures 2a, 2b, 2c, and 2d show the plot of the efficiency vs figure of merit at the photocathode, at the 12th dynode with high gain and with low gain. Figures 2a and 2b are for ^{3}H and 2c and 2d are for ^{14}C.

Figures 3a, 3b, 3c, and 3d plot the difference between the efficiency at the output of the photocathode and the efficiency at the output of the 12th dynode as a function of the figure of merit for high and low gain.

Table 3 shows the efficiency variation from the photocathode to the 12th dynode for different values of the figure of merit at low and high gain, for ^{3}H, for a single photomultiplier tube, and for two photomultiplier tubes working in coincidence.

Figure 4 plots the composed and the experimental efficiency vs quenching for ^{3}H and ^{14}C.

Table 4 contains the numerical values shown in the Figure 4.

Figure 5 shows the efficiency for low quantum yield at the photocathode for ^{14}C . The points are computed and the line corresponds to the experimental data.

The experimental values data have been obtained with a set of quenched standards for ^{3}H and ^{14}C, in a L.K.B. 12 19 Rackbeta spectral liquid scintillation spectrometer.

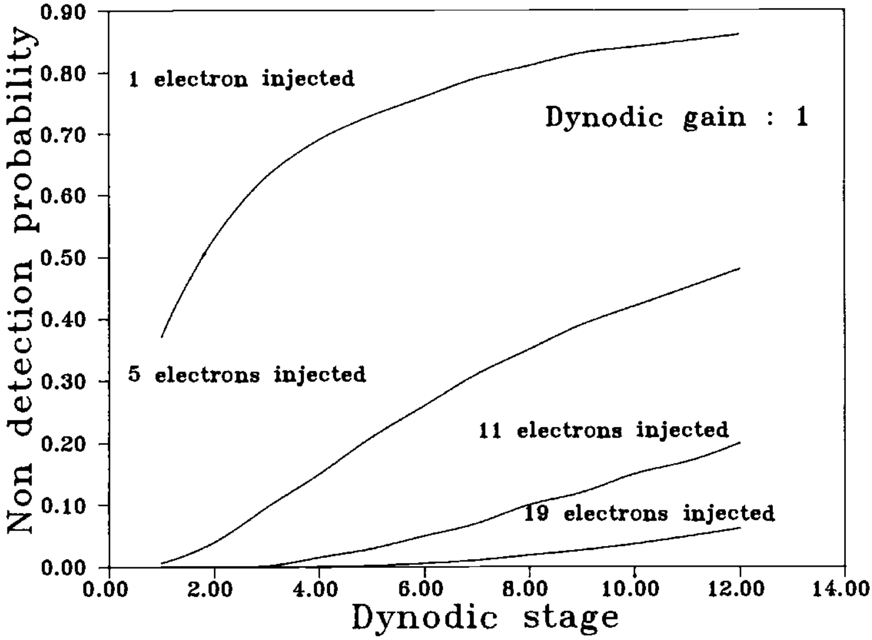

Figure 1a. Nondetection probability vs dynodic gain.

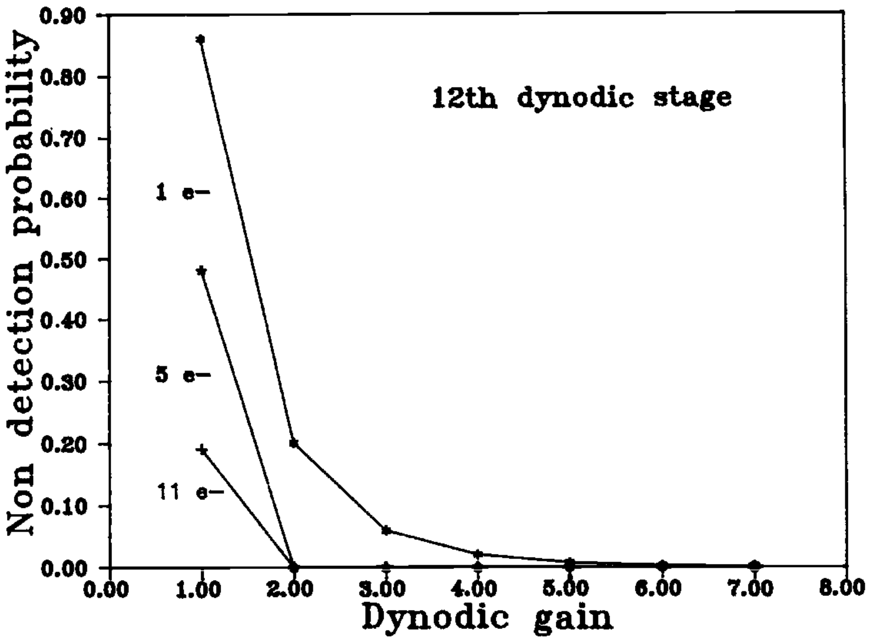

Figure 1b. Nondetection probability vs dynodic stage.

Table 2. Total LSC Efficiency Calculation for H-3 at the 12th Dynode, High Gain Two Photomultiplier Tubes in Coincidence

Fig. Merit	Efficiency (%)		Fig. Merit	Efficiency (%)		Fig. Merit	Efficiency (%)		Fig. Merit	Efficiency (%)
0.2500	84.093	*	12.7500	2.5803	*	25.2500	0.75114	*	37.7500	0.35151
0.5000	73.011	*	13.0000	2.4946	*	25.5000	0.73746	*	38.0000	0.34712
0.7500	63.529	*	13.2500	2.4131	*	25.7500	0.72417	*	38.2500	0.34278
1.0000	55.439	*	13.5000	2.3355	*	26.0000	0.71121	*	38.5000	0.33855
1.2500	48.576	*	13.7500	2.2616	*	26.2500	0.69862	*	38.7500	0.33436
1.5000	42.766	*	14.0000	2.1911	*	26.5000	0.68637	*	39.0000	0.33028
1.7500	37.845	*	14.2500	2.1238	*	26.7500	0.67441	*	39.2500	0.32628
2.0000	33.665	*	14.5000	2.0596	*	27.0000	0.66278	*	39.5000	0.32232
2.2500	30.100	*	14.7500	1.9983	*	27.2500	0.65142	*	39.7500	0.31846
2.5000	27.044	*	15.0000	1.9397	*	27.5000	0.64038	*	40.0000	0.31464
2.7500	24.411	*	15.2500	1.8836	*	27.7500	0.62958	*	40.2500	0.31092
3.0000	22.131	*	15.5000	1.8299	*	28.0000	0.61909	*	40.5000	0.30726
3.2500	20.146	*	15.7500	1.7784	*	28.2500	0.60886	*	40.7500	0.30363
3.5000	18.410	*	16.0000	1.7291	*	28.5000	0.59885	*	41.0000	0.30010
3.7500	16.883	*	16.2500	1.6818	*	28.7500	0.58911	*	41.2500	0.29660
4.0000	15.535	*	16.5000	1.6364	*	29.0000	0.57958	*	41.5000	0.29320
4.2500	14.339	*	16.7500	1.5928	*	29.2500	0.57031	*	41.7500	0.28985
4.5000	13.273	*	17.0000	1.5510	*	29.5000	0.56125	*	42.0000	0.28652
4.7500	12.320	*	17.2500	1.5107	*	29.7500	0.55238	*	42.2500	0.28329
5.0000	11.465	*	17.5000	1.4720	*	30.0000	0.54375	*	42.5000	0.28007
5.2500	10.695	*	17.7500	1.4348	*	30.2500	0.53529	*	42.7500	0.27694
5.5000	9.9984	*	18.0000	1.3990	*	30.5000	0.52706	*	43.0000	0.27387
5.7500	9.3673	*	18.2500	1.3645	*	30.7500	0.51898	*	43.2500	0.27081
6.0000	8.7936	*	18.5000	1.3312	*	31.0000	0.51111	*	43.5000	0.26784
6.2500	8.2704	*	18.7500	1.2992	*	31.2500	0.50343	*	43.7500	0.26488

6.5000	7.7923	*	19.0000	1.2683	*	31.5000	0.49588	*	44.0000	0.26200
6.7500	7.3541	*	19.2500	1.2384	*	31.7500	0.48853	*	44.2500	0.25917
7.0000	6.9515	*	19.5000	1.2097	*	32.0000	0.48132	*	44.5000	0.25636
7.2500	6.5809	*	19.7500	1.1818	*	32.2500	0.47429	*	44.7500	0.25362
7.5000	6.2390	*	20.0000	1.1550	*	32.5000	0.46742	*	45.0000	0.25089
7.7500	5.9228	*	20.2500	1.1291	*	32.7500	0.46066	*	45.2500	0.24823
8.0000	5.6300	*	20.5000	1.1040	*	33.0000	0.45408	*	45.5000	0.24562
8.2500	5.3582	*	20.7500	1.0798	*	33.2500	0.44760	*	45.7500	0.24302
8.5000	5.1056	*	21.0000	1.0563	*	33.5000	0.44130	*	46.0000	0.24049
8.7500	4.8703	*	21.2500	1.0336	*	33.7500	0.43510	*	46.2500	0.23797
9.0000	4.6508	*	21.5000	1.0116	*	34.0000	0.42906	*	46.5000	0.23552
9.2500	4.4458	*	21.7500	0.99030	*	34.2500	0.42314	*	46.7500	0.23310
9.5000	4.2540	*	22.0000	0.96964	*	34.5000	0.41731	*	47.0000	0.23070
9.7500	4.0743	*	22.2500	0.94966	*	34.7500	0.41163	*	47.2500	0.22836
10.0000	3.9057	*	22.5000	0.93026	*	35.0000	0.40603	*	47.5000	0.22602
10.2500	3.7473	*	22.7500	0.91148	*	35.2500	0.40059	*	47.7500	0.22375
10.5000	3.5983	*	23.0000	0.89326	*	35.5000	0.39525	*	48.0000	0.22152
10.7500	3.4580	*	23.2500	0.87555	*	35.7500	0.38998	*	48.2500	0.21928
11.0000	3.3257	*	23.5000	0.85839	*	36.0000	0.38485	*	48.5000	0.21711
11.2500	3.2009	*	23.7500	0.84169	*	36.2500	0.37978	*	48.7500	0.21495
11.5000	3.0829	*	24.0000	0.82551	*	36.5000	0.37485	*	49.0000	0.21284
11.7500	2.9713	*	24.2500	0.80976	*	36.7500	0.37002	*	49.2500	0.21077
12.0000	2.8657	*	24.5000	0.79449	*	37.0000	0.36524	*	49.5000	0.20869
12.2500	2.7655	*	24.7500	0.77965	*	37.2500	0.36059	*	49.7500	0.20668
12.5000	2.6705	*	25.0000	0.76518	*	37.5000	0.35599	*	50.0000	0.20466

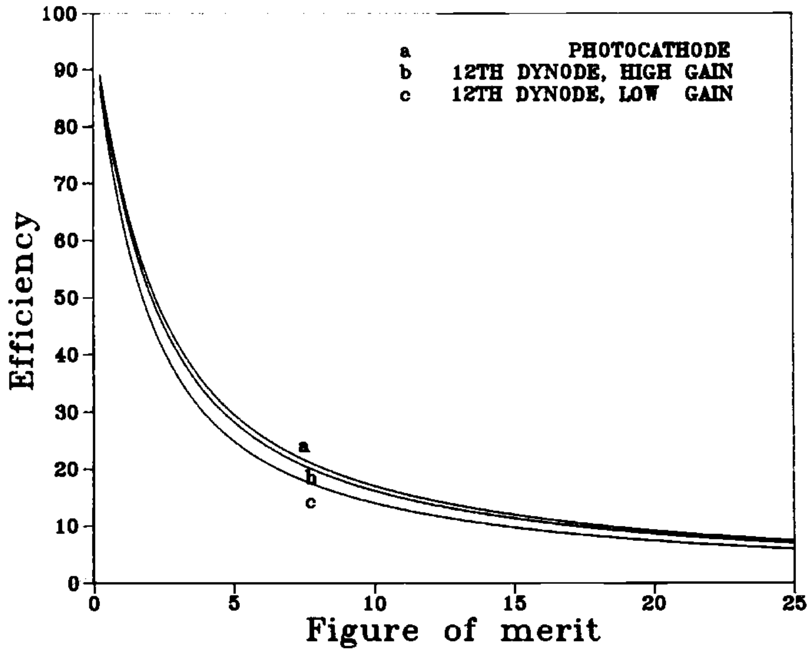

Figure 2a. ^3H: Efficiency vs Figure of merit, a single photomultiplier tube.

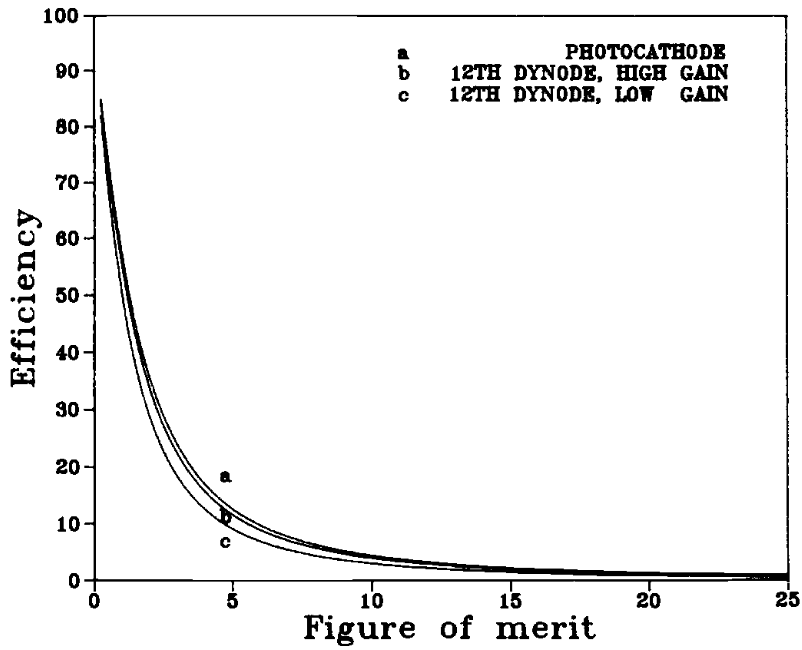

Figure 2b. ^3H: Efficiency vs Figure of merit, two photomultiplier tubes in coincidence.

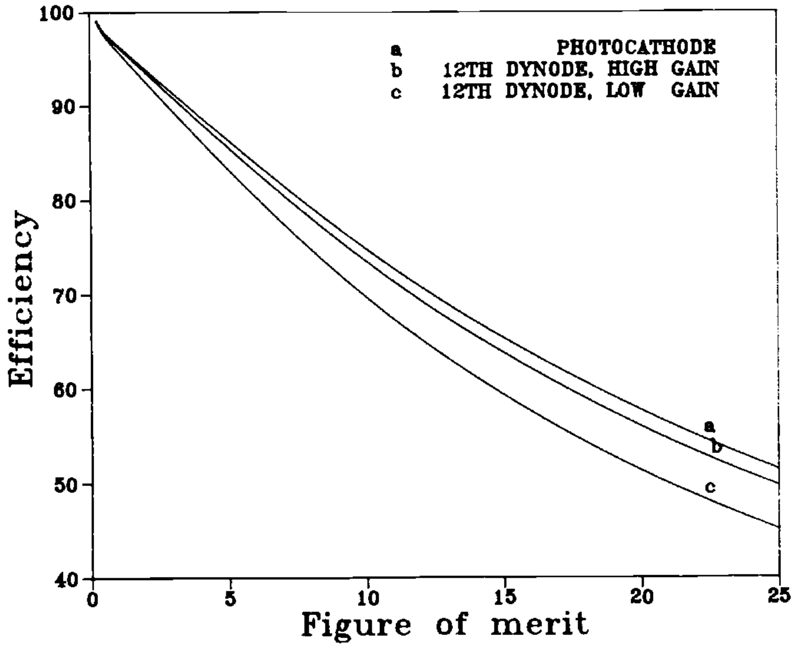

Figure 2c. ^{14}C: Efficiency vs Figure of merit, a single photomultiplier tube.

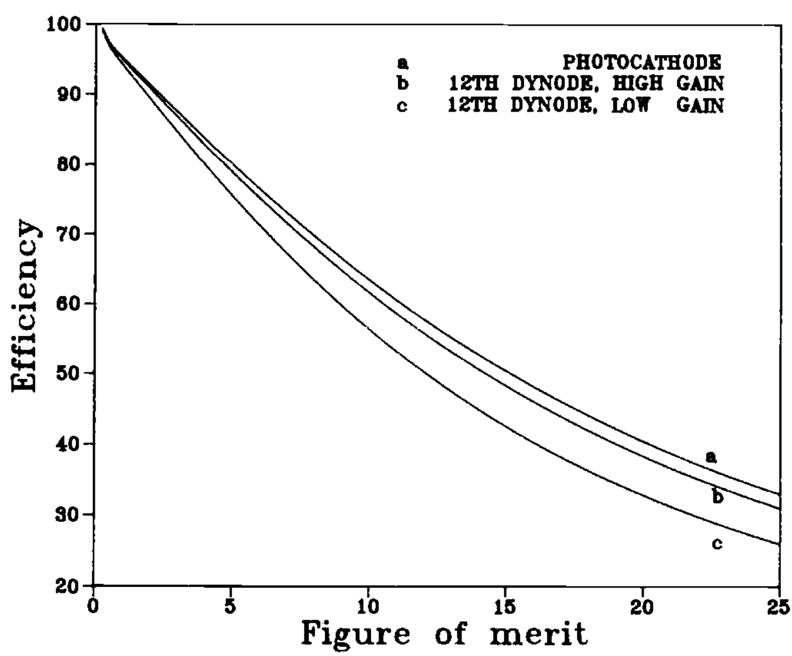

Figure 2d. ^{14}C: Efficiency vs Figure of merit, two photomultiplier tubes in coincidence.

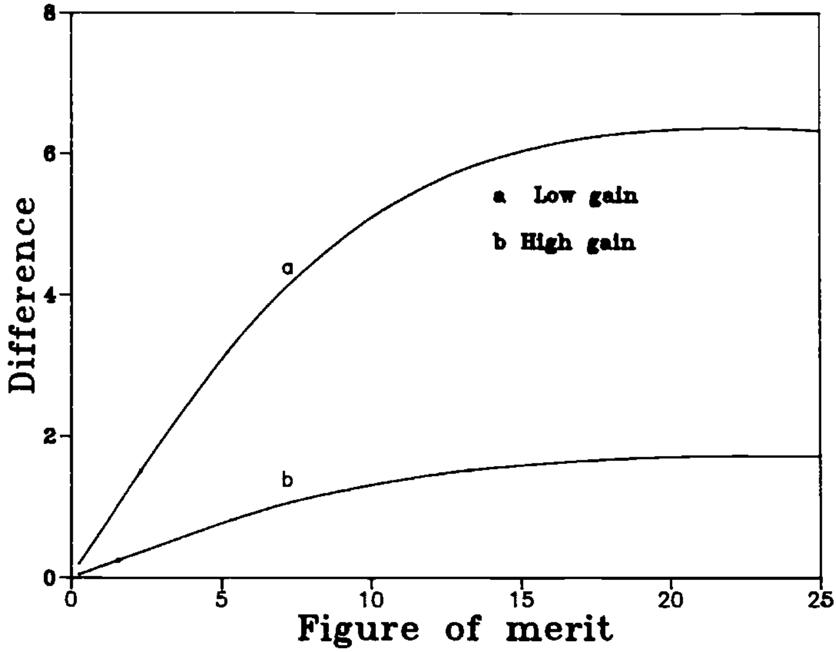

Figure 3a. Difference of efficiencies at the photocathode and at the 12th dynode for ^3H, a single photomultiplier tube.

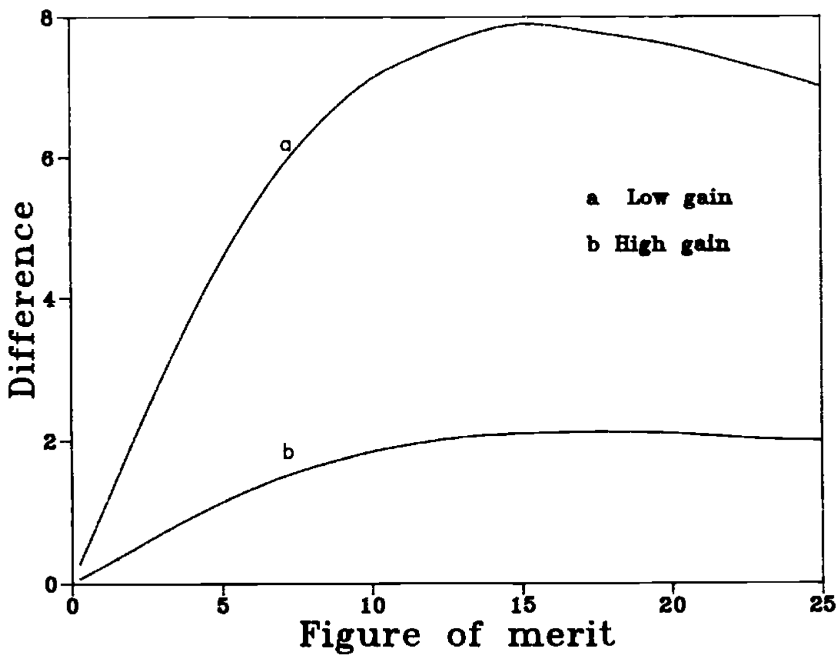

Figure 3b. Difference of efficiencies at the photocathode and at the 12th dynode for ^3H, two photomultiplier tube.

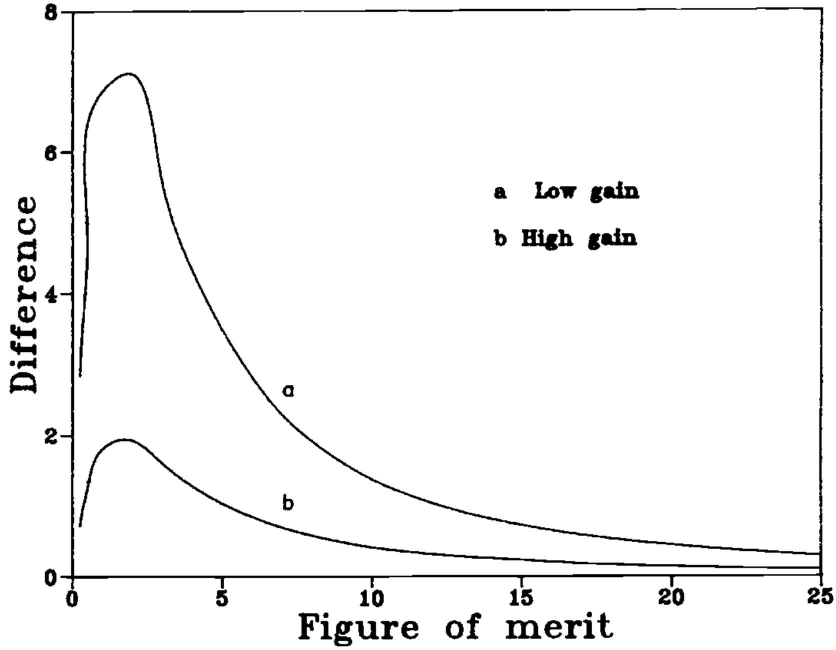

Figure 3c. Difference of efficiencies at the photocathode and at the 12th dynode for ^{14}C, a single photomultiplier tube.

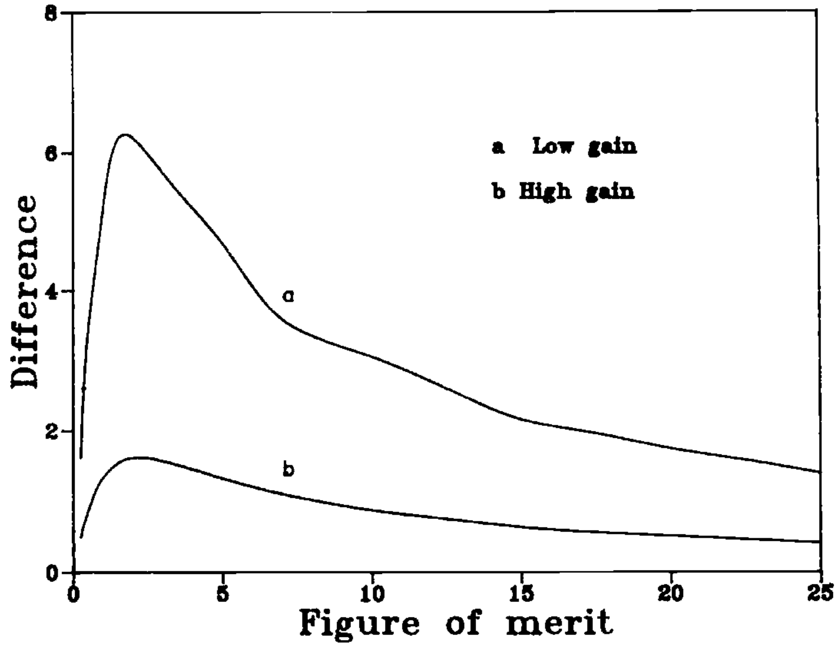

Figure 3d. Difference of efficiencies at the photocathode and at the 12th dynode for ^{14}C, two photomultiplier tube.

Table 3. Efficiency for ^{3}H in Photocathode and Successfive Dynodes

Two Photomultiplier Tubes in Coincidence						
STAGE	FOT.	2TH DY.	4TH DY.	6TH CY.	12TH DY.	
F.M.						
0.25	84.81	84.21	84.10	84.09	84.09*	
1.00	57.25	55.74	55.45	55.44	55.44*	
5.00	12.49	11.63	11.47	11.46	11.46*	
10.0	4.32	3.97	3.91	3.91	3.91*	HIGH GAIN
15.0	2.16	1.97	1.16	1.15	1.15*	
20.0	1.29	1.18	1.16	1.15	1.15*	
25.0	0.86	0.65	0.58	0.56	0.56*	
0.25	84.81	83.04	82.13	81.99	81.96*	
1.00	57.25	52.89	50.77	50.44	50.38*	
5.00	12.49	10.16	9.19	9.04	9.01*	
10.0	4.32	3.39	3.01	2.96	2.95*	LOW GAIN
15.0	2.16	1.66	1.47	1.45	1.44*	
20.0	1.29	0.99	0.87	0.86	0.85*	
25.0	0.86	0.65	0.58	0.56	0.56*	

A Single Photomultiplier Tube						
STAGE	FOT.	2TH DY.	4TH DY.	6TH CY.	12TH DY.	
F.M.						
0.25	89.03	88.61	88.53	88.53	88.53*	
1.00	69.44	68.33	68.11	68.11	68.10*	
5.00	29.54	28.44	28.23	28.22	28.22*	
10.0	16.93	16.21	16.07	16.06	16.06*	HIGH GAIN
15.0	11.85	11.31	11.21	11.21	11.21*	
20.0	9.11	8.69	8.61	8.60	8.60*	
25.0	7.39	7.05	6.98	6.98	6.98*	
0.25	89.03	87.80	87.16	87.06	87.04*	
1.00	69.44	66.20	64.60	64.35	64.30*	
5.00	29.54	26.47	25.09	24.89	24.85*	
10.0	16.93	14.23	14.05	13.92	13.89*	LOW GAIN
15.0	11.85	10.38	9.75	9.65	9.63*	
20.0	9.11	7.95	7.46	7.38	7.37*	
25.0	7.39	6.44	6.04	5.98	5.97*	

Table 4. Quenching Curve for ^{14}C Theoretical Values

		FIGURE OF MERIT			EFFICIENCY FOR ^{14}C		
Q(E)	EFFICIENCY 3H	FOT.	H.G.	L.G.	FOT.	H.G.	L.W.
451.61	58.33	0.97	0.91	0.77	95.23	95.24	95.25
434.61	54.18	1.11	1.04	0.89	94.67	94.69	94.66
394.12	47.15	1.40	1.31	1.11	93.54	93.57	93.57
351.88	38.05	1.85	1.74	1.47	91.79	91.79	91.82
309.94	27.41	2.61	2.47	2.10	88.88	88.82	88.79
250.72	15.86	4.19	3.94	3.33	82.95	82.95	82.99
206.16	09.03	6.27	5.90	4.99	75.47	75.45	75.49

PHO: Photocathode; H.G.: 12th Dynode high gain; L.W.: 12th Dynode low gain

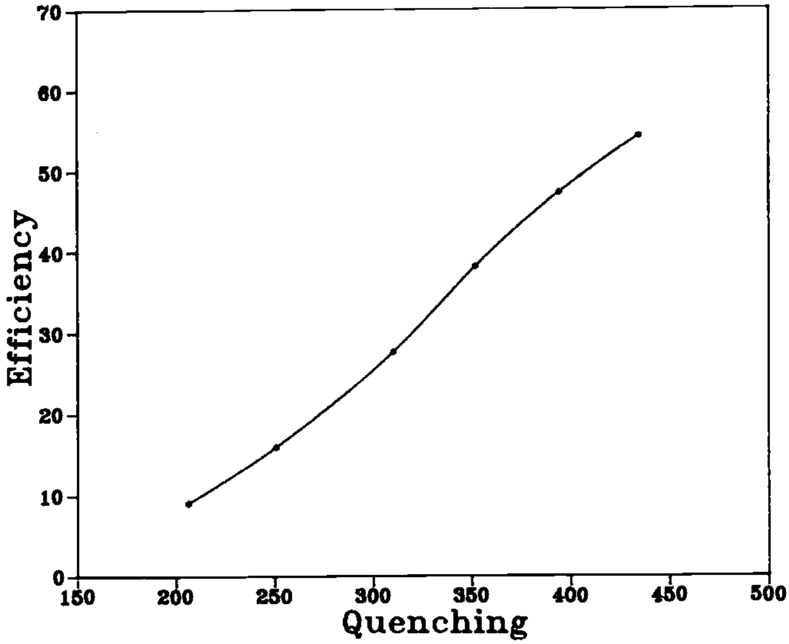

Figure 4a. Quenching curve for ^3H.

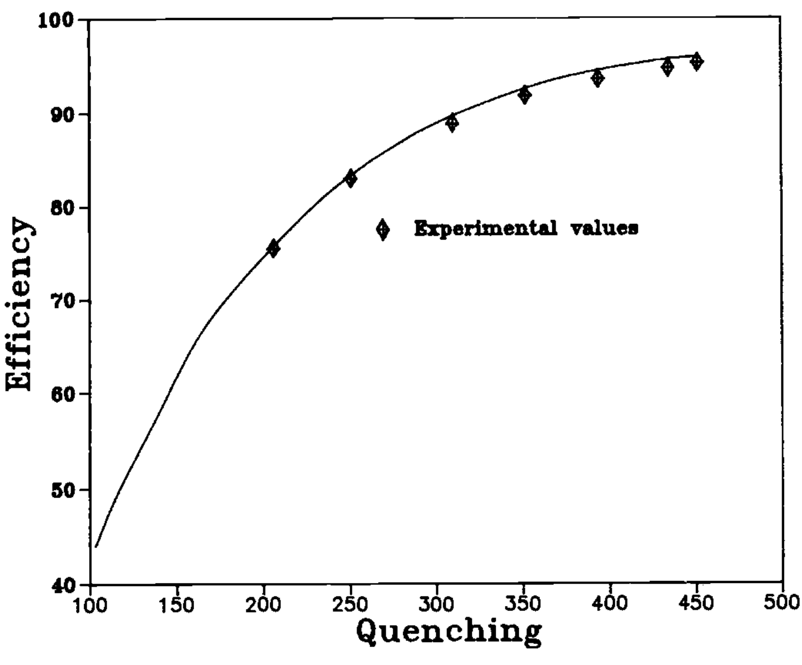

Figure 4b. Quenching curve for ^{14}C, experimental and calculated.

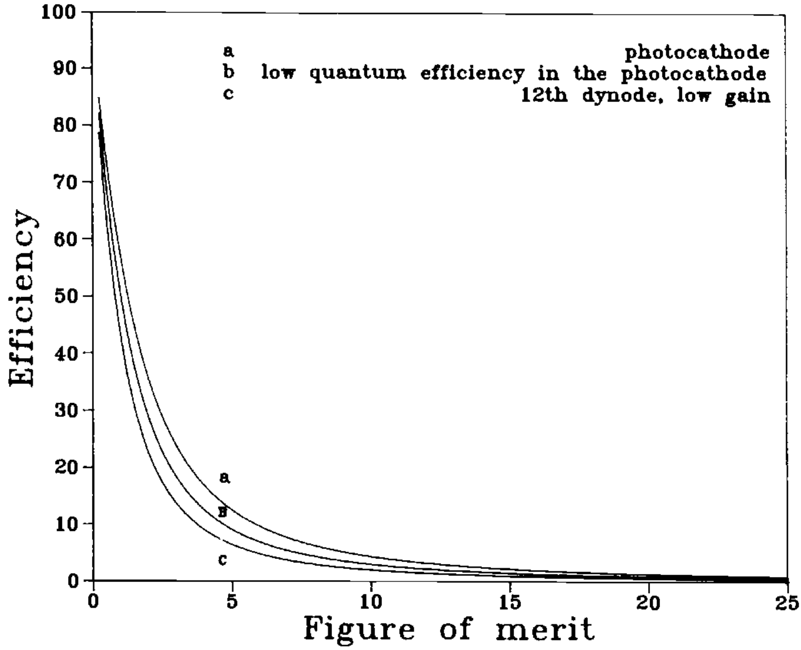

Figure 5a. ^{3}H efficiency vs Figure of merit, two photomultiplier tubes in coincidence, low quantum efficiency at the photocathode.

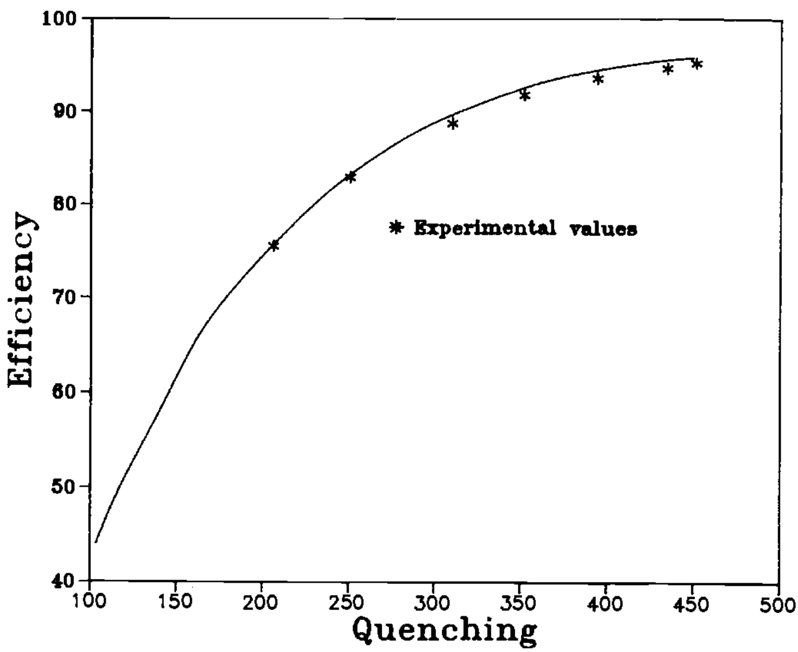

Figure 5b. Quenching curve for ^{14}C, experimental and calculated, low quantum efficiency at the photocathode.

DISCUSSION

The results of the previous section show that the nondetection probability falls down very quickly when the number of electrons reaching the first dynode grows. It also falls with the increase of the dynodic gain. However, this nondetection probability is greater for the latest dynodic stages.

The model developed allows efficiency computation in any dynodic stage that is lower than the efficiency at the photocathode, as a function of the figure of merit for a single photomultiplier or for a two photomultiplier system working in coincidence. The computed efficiencies do not vary significantly after the 4th or 5th dynodic stage.

The model permits obtaining the quenching curve of a nuclide from the experimental quenching curve of another nuclide. It does this by using the figure of merit efficiency function for both nuclides, at the photocathode or at the output of any dynodic stage. The computed values are the same regardless of the case considered.

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